Stochastic resonance in small-particle magnetics. I. Radiospectroscopic study

1992 J. Phys.: Condens. Matter 4 3295

(http://iopscience.iop.org/0953-8984/4/12/020)

J. Phys.: Condens. Matter 4 (1992) 3295–3298. Printed in the UK

Stochastic resonance in small-particle magnetics: I. Radiospectroscopic study

E K Sadykov
Department of Physics, Kazan State University, 420008 Kazan, Russia

Received 15 July 1991, in final form 25 November 1991

Abstract. The magnetic susceptibility of the superparamagnetic particle modulated along its magnetic easy axis by the RF field has been calculated. The temperature behaviour of the pole of this function at the modulation frequency may be considered as a realization of the stochastic resonance in this system.

1. Introduction

The stochastic resonance (SR) in a stochastic bistable system subjected to a small external signal has been investigated in recent years [1–4]. This phenomenon consists of a 'resonant' dependence (preliminary increase and subsequent montonic decrease) of the output signal intensity (or signal-to-noise relation in the output) at the modulation frequency on the input noise intensity of a stochastic bistable system. More physical justification of the SR notion is that the maximum in an output signal is achieved on coincidence of the modulation frequency with the mean rate of the stochastic transitions between two stable states of the bistable system. Many workers have interpreted these results as a cooperative effect caused by the combination of the stochastic non-linear dynamics of the system and the external periodic force. However, an explanation of the SR using linear response theory was also proposed in [5].

2. Theoretical considerations

In this paper we consider the superparamagnetic particle (SPP) driven along its magnetic easy axis by an RF field as a modulated stochastic bistable system. In such a system the RF field and the temperature serve as the input signal and the input noise, respectively. The autocorrelation function and its spectral density (SD) for the particle magnetization are of interest from a physical point of view. As will be seen later, the SD has a spike at the modulation frequency which may be considered as an output signal. Let us define also as the output noise the value of SD in the vicinity of this pole. In a recent paper [6] the Mössbauer spectroscopy method for experimental study of the SPP in the conditions under discussion was proposed. Here we discuss the possibility of a radiospectroscopic study of the SR in this system. For this reason, one has to calculate the magnetic susceptibility by assuming its relation to the SD in the sense of the fluctuation–dissipation theorem.

Usually the theoretical consideration of the SR is founded on the one-dimensional Langevin equation for the overdamped motion in a bistable potential and its Fokker–Planck approach. In our case we deal with the two-dimensional Gilbert equation for the SPP magnetization (the value of the magnetization is believed to be constant [7, 9]. Only the longitudinal (along the easy axis) magnetization of the particle obeys the equation of overdamped motion and gives us a chance to observe the SR. Thus the Fokker–Planck equation for the SPP must be taken in the form

$$\partial P(x,t)/\partial t = (L_0 + L_{\mathrm{m}} + L_{\mathrm{pr}})P(x,t) \tag{1}$$

where

$$L_0 = -\nu(\partial/\partial x)\{(1-x^2)[2Kx - (k_{\mathrm{B}}T/v)(\partial/\partial x)]\} \tag{2}$$

is an unperturbed Fokker–Planck operator for the SPP in the uniaxial potential of the magnetic anisotropy [7], L_{m} and L_{pr} represent the modulating and probe RF fields:

$$L_{\mathrm{m}} = -\nu M_{\mathrm{s}} H_{\mathrm{m}} \cos(\omega_{\mathrm{m}} t + \varphi_{\mathrm{m}})(\partial/\partial x)(1-x^2)$$

$\nu = \eta\gamma^2/(1+(\eta\gamma M_{\mathrm{s}})^2)$, M_{s}, γ and η are the magnetization, the gyromagnetic ratio and the damping constant, respectively, H_{m}, ω_{m} and φ_{m} are the amplitude, the frequency and the phase of the RF field, K and v are the uniaxial anisotropy constant and the volume of the particle, and $x = \cos\theta$, where θ is the polar angle of the SPP magnetic moment relative to the easy axis. If we need a longitudinal susceptibility, L_{pr} has the same form as L_{m} with the subscripts m replaced by subscripts pr.

Equation (1) differs from that used in the theory of linear susceptibility [8] in that the 'unperturbed' Fokker–Planck operator now includes the time-dependent term L_{m}. Nevertheless we follow this theory using a solution of the equation

$$(L_0 + L_{\mathrm{m}})P_{\mathrm{m}}(x,t) = \partial P_m(x,t)/\partial t \simeq 0 \tag{3}$$

as the 'unperturbed' situation. The approximation accepted in (3) is justified in the limit $\omega_m \ll 2K\nu$ (this condition is the same as the condition of the adiabatic approach in the theory of the SR [3]). Omitting some details of the calculations we present the required expression for the imaginary part of the longitudinal magnetic susceptibility:

$$\chi''(\omega) = \frac{vM_{\mathrm{s}}^2\omega}{2k_{\mathrm{B}}T}\int_{-\infty}^{\infty} \mathrm{d}\tau \exp(-\mathrm{i}\omega\tau)\,\langle x(t+\tau)x(t)\rangle_t \tag{4}$$

ω is the frequency of the probe field. The autocorrelation function is generalized for the quasistationary state $P_{\mathrm{m}}(x,t)$:

$$\langle x(t+\tau)x(t)\rangle = \int\!\!\int \mathrm{d}x\,\mathrm{d}x_0\,\hat{T}\exp\left(\int_t^{t+\tau}[L_0 + L_{\mathrm{m}}(t')]\,\mathrm{d}t'\right)x_0 P_{\mathrm{m}}(x,t)$$

and averaged over the modulation period. The expression for the autocorrelation function and its SD for the bistable system under external modulation in the framework of the discrete orientation model and in the linear (relative to the RF field) approximation were obtained in [3]. Using these results we have

$$\chi''(\omega) = [vM_{\mathrm{s}}^2\omega W_0/k_{\mathrm{B}}T(W_0^2+\omega^2)][1 - W_1^2A^2/2(W_0^2+\omega_{\mathrm{m}}^2)] + [vM_{\mathrm{s}}^2\omega\pi W_1^2A^2/2k_{\mathrm{B}}T(W_0^2+\omega^2)]\delta(\omega-\omega_{\mathrm{m}}) \tag{5}$$

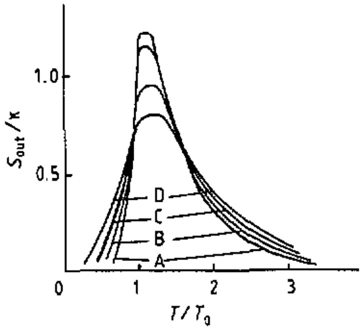

Figure 1. The SR curves S_{out}/κ averaged for various values of D: curve A, $D = 0.01$; curve B, $D = 0.1$; curve C, $D = 0.2$; curve D, $D = 0.3$.

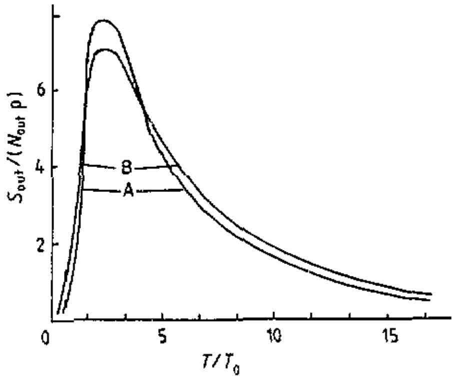

Figure 2. The SR curves $S_{out}/N_{out}\rho$ averaged for various values of D: curve A, $D = 0.01$; curve B, $D = 0.3$.

$$W_0 = W_1 = \alpha_0 \exp(-\beta) \qquad \alpha_0 = 2\eta K/[1 + (\gamma\eta M_s)^2]$$
$$\beta = vK/k_B T \qquad A = vM_s H_m/k_B T.$$

The linear approximation used here is in accordance with the linear response conception in the theory of the SR [5]. Equation (5) differs from the expression (at the $\beta \gg 1$ limit) in [9] in that it has an additional δ-function term which is caused by the modulation process in the stochastic system. Let us consider the peculiarities of the dependence of this term (output signal) on T, ω_m and H_m:

$$S_{out} = vM_s^2\omega_m W_1^2 A^2/2k_B T(W_0^2 + \omega_m^2)\sigma. \tag{6}$$

Define also the output noise at modulation frequency (the first term in (5) at $\omega = \omega_m$) by

$$N_{out} = [vM_s^2\omega_m W_0/k_B T(W_0^2 + \omega_m^2)][1 - W_1^2 A^2/2(W_0^2 + \omega_m^2)] \tag{7}$$

and a signal-to-noise relation as

$$S_{out}/N_{out} = (W_1 A^2/2\sigma)[1 - W_1^2 A^2/2(W_0^2 + \omega_m^2)]^{-1}. \tag{8}$$

In equations (6) and (8) the finite bandwidth σ is introduced which is actually realized in experiment. We see (figure 1(A)) that S_{out} increases at first as T increases (input noise) and falls gradually to zero for large values of T. This behaviour is exactly the same as that for SR [4]. Qualitatively S_{out}/N_{out} has the same dependence on T with a maximum displaced to larger values of T (figure 2 (A)).

The normalization temperature T_0 in figures 1 and 2 is introduced by the relation $\alpha_0 \exp(-vK/k_B T_0) \simeq \omega_m$ with $\alpha_0 \simeq 10^{10}\,s^{-1}$. At room temperature ($T_0 \simeq 300$ K) and $\omega_m \simeq 10^8\,s^{-1}$, for example, the volume of the SPP is $v_0 \simeq 10^3\,nm^3$ for $K = 4 \times 10^4\,J\,m^{-2}$ (iron sample). The estimation on the basis of (8) gives us $S_{out}/N_{out} > 1$ for the RF amplitudes H of the order 10^{-4} Tl ($\sigma \simeq 10^4\,s^{-1}$ was used in this estimation). The H_m and ω_m dependences of S_{out} and S_{out}/N_{out} are evident also from (6) and (8).

The results need to be corrected because of the distribution of the SPP sizes in real systems. The curves in figures 1 and 2 are obtained by averaging equations (6) and (8) using the normal size distribution:

$$f(v) = (1/\Delta v\sqrt{2\pi})\exp[-(v-v_0)^2/2(\Delta v)^2]$$

with $\Delta v = v_0 D$. The coefficients κ and ρ in figures 1 and 2 are of the forms

$$\kappa = \omega_m M_s^4 H_m^2[\ln(\alpha_0/\omega_m)]^3/2\sigma K^3\sqrt{2\pi} \qquad \rho = K\kappa/M_s^2\ln(\alpha_0/\omega_m).$$

Note that S_{out} must be multiplied by the 'diluting factor' equal to the relative volume of the SPP in substance.

An essential smoothing out of the 'resonance' for S_{out} takes place for large values of D. Consequently the sharpness of this 'resonance' reflects the homogeneity of the system relative to the SPP volume. Some corrections are needed to (6) and (8) also owing to the chaotic distribution of the easy axes of the magnetic particles in real systems.

3. Discussion

Thus a system of non-interacting magnetic particles may be used for experimental studies of the SR using the well known radiospectroscopic method. Moreover we believe this phenomenon to be fruitful in the investigation of the problems of finely dispersed magnetism. For example the high-frequency magnetization reversal process of these substances in the regime of their bistability may be of interest. As a direct candidate for experiments the metallopolymer systems based on Fe, Cr, Co and Ni [10] may be suggested.

In conclusion we expect the SR to be observable in finely dispersed magnetics over a wide range of particle sizes. An expression of the type $\alpha_0\exp(-\beta) \sim \omega_m$ may serve as a guiding relation for the estimation of the optimal experimental conditions.

References

[1] Presilla S, Marchesoni F and Gammaitoni L 1989 *Phys. Rev.* A **40** 2105
[2] Gammaitoni L, Menichella-Saetta F, Santucci S, Marchesoni F and Presilla C 1989 *Phys. Rev.* A **40** 2114
[3] McNamara B and Wiesenfeld K 1989 *Phys. Rev.* A **39** 4854
[4] Debnath G, Zhou T and Moss F 1989 *Phys. Rev.* A **39** 4323
[5] Dykman M I, Mannela R, McClintock P V E and Stocks N G 1990 *Phys. Rev. Lett.* **65** 2606
[6] Sadykov E K and Skvortsov A I 1990 *Zh. Eksp. Teor. Fiz. Pis. Red.* **52** 752
[7] Brown W F Jr 1963 *Phys. Rev.* **130** 1677
[8] Agarwal G A S 1972 *Z. Phys.* **252** 25
[9] Raikher Ju L and Shliomis M I 1974 *Zh. Eksp. Teor. Fiz.* **67** 1060
[10] Petrakovskii G A, Aplesnin S S and Piskorsky V P 1984 *Izv. Vuzov, Phiz.* No 10 46